Aswers to Seminar 2 ECON4930 Tuesday 15 february

1. **Hveding's conjecture**:
The drivers for price change in the aggregate model (figure 3.10, and related text) (basic model in Lecture 1,2, figure in Lecture 3):
i) Lower constraint on the reservoir binding, i.e. emptying the reservoir
Must start using the backward principle at time T, emptying is implied by the assumption of positive production in every period and no satiation, i.e. $p_t > 0$ for all t. From the first –order conditions we then have $p_T = \lambda_T > 0$

ii) Going backwards it seems reasonable to introduce a possibility for a second period, t+1,of emptying the reservoir; we have a last cold winter spell before inflows are sufficient for matching current demand. From the first –order conditions we then have for periods t+1 and t (remember that in the aggregate model we alwys have prie equal to water value):
$-\lambda_{t+1} + \lambda_{t+2} - \gamma_{t+1} \le 0 \ (R_{t+1} = 0)$
Assume $\gamma_{t+1} = 0$, then $\lambda_{t+2} \le \lambda_{t+1}$. Typically, we have $\lambda_{t+1} > \lambda_{t+2}$. Furtermore, if we assume $\gamma_t = 0$, we then have $-\lambda_t + \lambda_{t+1} = 0 \ (R_t > 0) \Rightarrow \lambda_t = \lambda_{t+1}$, i.e. the price in period t is the same price as in period t+1.

iii) Assume threat of overflow in period s, but not in period s+1 or in period s-1. We then have the price from the future: $p_{s+1} = \lambda_{s+1} = \lambda_{t+1}$. Reservoir dynamics yield $-\lambda_s + \lambda_{s+1} - \gamma_s = 0 \ (R_s > 0)$. Typically we then have $\gamma_s > 0$ and
$\lambda_s = \lambda_{s+1} - \gamma_s \Rightarrow \lambda_{s-1} = \lambda_s < \lambda_{s+1}$

Multiple plants:

Situation at t=T: all reservoirs must be emptied, we still assume positive period prices, from the first-order conditions we have $p_T = \lambda_{iT} > 0 (i = 1,..,N)$. All the plants must have the same water values at period T due to electricity being a perfectly homogeneous good.

Going backwards to period t+1: Let us assume that $p_{t+1}>p_{t+2}$, is it then possible that plant i is not emptying its reservoir in period t+1 provided that it is physically possible to do so? If it is emptying its reservoir the first-order condition is:
$-\lambda_{i,t+1} + \lambda_{i,t+2} - \gamma_{i,t+1} \le 0 \ (R_{i,t+1} > 0) \Rightarrow \lambda_{i,t+1} = \lambda_{i,t+2} - \gamma_{i,t+1}$. But this is a contradiction both if γ_{is} >0 and = 0, with our asssumptions about the relationship between the prices, $p_{t+1}>p_{t+2}$. Therefore, the reservoir must be emptied.

Going backwards to period s, and assume $p_s< p_{s+1}$. Is it then possible that reservoir i is not full in period s? In that case the first-order condition is
$-\lambda_{is} + \lambda_{i,s+1} - \gamma_{is} \le 0 \ (R_{is} \ge 0) \Rightarrow \lambda_{is} \ge \lambda_{i,s+1}$. We have by assumption that γ_{is} =0. But then we have a contradiction both if R_{is} > 0 and = 0, to the relationship between the prices,

i.e. reservoir i has to be full at the end of period s in order to benefit fully from the higher price in period s+1 by transferring the maximal amount of water.

Notice that when we have a period with a threat of overflow, then we have $-\lambda_{is}+\lambda_{i,s+1}-\gamma_{i,s}=0$ for all reservoirs and period s and s+1 prices being equal to the corresponding plant water values.This implies that all the individual reservoir limit shadow prices must be equal. One m^3 more water measured in kwh (NB! Of crucial importance) adds the same amount to the objective function irrespective which dam we enlarge.

Take any periods u, u+1 within a subperiod of periods when the prices are equal. Is it then possible that there is a threat of overflow at reservoir i? In that case we must have $-\lambda_{is}+\lambda_{i,s+1}-\gamma_{i,s}=0\ (R_{is}>0)\Rightarrow\lambda_{is}=\lambda_{i,s+1}-\gamma_{i,s}$. But since the period prices are equal, we must have $\gamma_{i,s}=0$. So it is possible to have a threat of overflow, but then the shadow price on the reservoir consraint must be zero. This also holds for emptying the reservor in such a period with equal prices: the water values must remain equal for the periods. This is the fundamental indeterminacy of utilisation of reservoirs within periods with the same price.

All that matters for optimality is that the reservoirs are emptied at the same time when the prices change, and that we have threat of overflow at the same time when the price change.

The electricity generation and the reservoir capacities can be added together because the water values and shadow prices are all the same when the price changes.

2. Multi-year reservoir

A single aggregated system cannot be used to tell us when a multi-year reservoir will be utilised. A multi-year reservoir is so big that all water can be saved to the period with the highest price. For this period the aggregate model will be correct if we use the maximal amount of water accumulated during the T periods and not the physical limit, but the aggregate model will not be correct if the multi-year reservoir is larger than marginal in the model. This means that adding inflows to this reservoir to the total inflow may in principle lead to a different solution for all periods.

My position now is that Hveding's conjecture only holds if the reservoirs are small enough that all inflows for all periods cannot be transferred to the highest price period. So the aggreation problem remains even if we do not have a multi-year reservoir, but just a reservoir that is big enough for all inflows to tbe transferred to the highest price period. The point is that when all reservoirs go in step when prices change, then the water values and the shadow prices are equal for a time period, but for accumulating reservoirs the water value is equal to the price in the period the water is finally used for all periods before this period. Such cases cannot be revealed by the aggregate model.

3. Extra question

The constraint for the reservoir for the terminal period T is $R_T \geq \underline{R}_T \Rightarrow -R_T \leq -\underline{R}_T$. Therefore, the new condition in the Lagrangian becomes $-\varphi_T(-R_T + \underline{R}_T)$. The first-order condition for the reservoir level at the end of period T becomes:

$\frac{\partial L}{\partial R_T} = -\lambda_T + \varphi_T = 0 \Rightarrow \lambda_T = \varphi_T$. The water value in the terminal period becomes equal to the shadow price on the level of the terminal reservoir constraint, and it has to be binding because demand for electricity is not satiated. From the envelope theorem:

$\frac{\partial(\text{objective function})}{\partial \underline{R}_T} = \frac{\partial L}{\partial \underline{R}_T} = -\varphi_T$. The more water we want to leave after T, the lower the value of the objective function becomes.

4. The constraint for a period t_o: $R_{t_o} \geq \underline{R}_{t_o} \Rightarrow -R_{t_o} \leq -\underline{R}_{t_o}$. The new condition in the Lagrangian: $-\varphi_{t_o}(-R_{t_o} + \underline{R}_{t_o})$. Assuming now that the prices around t_o are equal (and for instance equal to the price at t+1 when the reservoir is emptied for the last time before the terminal period) the first–order condition for the reservoir level for period t_o is $-\lambda_{t_o} + \varphi_{t_o} + \lambda_{t_o+1} = 0\ (R_{t_o} > 0) \Rightarrow \lambda_{t_o} = \lambda_{t_o+1} + \varphi_{t_o}$. When the new constraint in period t_o is binding then the price in period to typically becomes greater than the previous price. If before the constraint was introduced the prices where equal after period t+1, then we have a new higher price from period t_o and backwards to period 1. This cannot be optimal for the social planner! Since more water is available after period t_o the price level must be lower in the periods after than before the constraint until the reservoir is emptied in some period before T, but this period may now be different from period t+1, and if so the terminal price may also be different. But the price will be the same if no more water is accumulated after the episode with emptying the reservoir.

With the reservoir level constraint the condumers pay a higher price for all earlier periods and a lower price for later perids. It is as if the consumers are forced to pay an insurance premium. But in a deterministic world imposing such a constraint reduces the value of the objective function.